Nonlinear Aeroservoelastic Control in the Presence of Uncertainty

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This work considers the development of two new strategies for the suppression of limit cycle oscillations in aeroservoelastic systems with uncertain structural nonlinearities. The first method features an adaptive linearization approach in which a compactly-supported radial basis function network is used to approximate nonlinearities. The functional form of the nonlinearities does not need to be known a priori in this approach. Moreover, the compact-support of the radial basis functions allows for the extension of adaptivity to nonlinearities with discontinuities. The second method uses a novel dissipativity-based control technique that makes both the linear and nonlinear subsystems dissipative. Therefore, the system is able only to dissipate energy and thus its behaviour is asymptotically stable. The main advantage of this method is that the control technique is capable of stabilizing the system even in presence of an error in the cancellation of the nonlinearity. Both methods are tested numerically and applied on an experimental aeroservoelastic system in the final version of this paper.

I. Introduction

A. Background

In recent years, the advent of new composite materials with high specific strengths has given impetus to the design and development of lightweight, flexible aerostructures. Such designs enjoy the benefits of increased aerodynamic, fuel and environmental efficiency. However, they can also give rise to undesired nonlinear dynamic behaviour. One example is that of limit cycle oscillations (LCOs) which, if uncontrolled, can lead to reduced fatigue life of an aircraft or even catastrophic structural failure.

The use of active control to suppress LCOs has been investigated both numerically and experimentally by a plethora of authors [1]. Most commonly, feedback linearization is used, where the nonlinearity is cancelled by means of a control input such as a leading or trailing-edge control surface.

Starting from the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u \tag{1}$$

$$y = \mathbf{h}(\mathbf{x}) \tag{2}$$

where $\mathbf{x} \in \mathbb{R}^{n_x}$ is the state variable, $u \in \mathbb{R}^{n_u}$ is the control input and $y \in \mathbb{R}^{n_y}$ represent the measured output, the input-output linearization method is based on the idea of linearizing the map between some transformed input v to the actual output y. In order to do that, one has to impose an input u to the system so that it is capable of exactly cancelling the nonlinearity. It may be shown that the control input is of the form

$$u = \frac{v - \alpha(\mathbf{x})}{\beta(\mathbf{x})} \tag{3}$$

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where $\alpha(\mathbf{x})$ and $\beta(\mathbf{x})$ are nonlinear cancellation terms, and v is the equivalent linear input. The linear control input v is designed for the resulting linearized input-output model using standard techniques such as pole placement.

The conventional feedback linearization approach requires that the form of the nonlinearity is known exactly a priori. In practice, this is difficult to achieve since knowledge of the nonlinearity may be limited or, in the worst-case, the nonlinearity itself may act at a location in the system that is not observable. Consequently, the true input to the system is merely an estimate of the form

$$u = \frac{v - \hat{\alpha}(\mathbf{x})}{\hat{\beta}(\mathbf{x})} \tag{4}$$

where the $\hat{\cdot}$ operator denotes the estimation of the function.

Due to estimation error, unintended dynamics may be introduced when the feedback controller is applied to the system. In the worst case, it is possible that the system is driven into instability.

Motivated by this problem, this work considers active LCO suppression using both: i) a new adaptive feedback linearization approach that is capable of cancelling nonsmooth nonlinearities of unknown functional form and ii) a novel dissipativity based control method. Both methods consider the problem of inexact cancellation of nonlinearities and are benchmarked against the standard feedback linearization approach.

II. Adaptive Feedback Linearization using Radial-Basis Functions

A well-known solution to the abovementioned problem is to couple feedback linearization with adaptive control techniques [2]. In this approach, the nonlinearity is approximated as a weighted sum of contributory functions

$$f_{\mathrm{NL}}(x, \boldsymbol{\theta}) = \sum_{i=1}^{p} \boldsymbol{\theta}_i f_i(x) \tag{5}$$

so that the corresponding closed loop system becomes

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \tilde{\boldsymbol{\theta}}\mathbf{r}\mathbf{b} \tag{6}$$

where $\tilde{\boldsymbol{\theta}}$ is the vector of errors between the true and estimated parameters, and $\mathbf{r}$ and $\mathbf{b}$ are terms corresponding to the assumed functional form of the nonlinearity in Eq. 5. The parameters are updated in real-time according to the law

$$\dot{\boldsymbol{\theta}} = \boldsymbol{\Gamma}\mathbf{r}\mathbf{b}^T\mathbf{P}\mathbf{z} \tag{7}$$

where $\boldsymbol{\Gamma}$ and $\mathbf{P}$ are positive definite matrices that serve to adjust the update rate of the parameters. Eq. 7 is chosen so that the system remains stable in the Lyapunov sense [3].

The conventional adaptive feedback linearization approach requires that the functional form of the nonlinearity is either: i) known exactly, or ii) can be well approximated by continuous basis functions. For example, it is common to approximate nonlinearities by polynomial fittings so that

$$f_{\mathrm{NL}}(x) \approx \sum_{i=0}^{p} a_i x^i \tag{8}$$

Although shown to serve well for geometric nonlinearities, this approximation is usually unsuitable for nonlinearities with non-negligible discontinuities. Moreover, the approximation of a nonlinearity by a polynomial basis may have little physical justification and therefore other bases could be more suitable.

In this work, the adaptive feedback linearization approach is generalized to systems that contain discontinuous non-linearities. To do this, the nonlinearity is expressed as the sum of compactly-supported radial basis functions, so that

$$f_{\mathrm{NL}}(x, \boldsymbol{\theta}) = \sum_{i=1}^{p} \theta_i \psi_i(||x - \hat{x}_i||) \tag{9}$$

where

$$\psi_i(||x - \hat{x}_i||) = \begin{cases} 1 & \text{if } ||x - \hat{x}_i|| < R_i, \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

This is equivalent to modelling the nonlinearity as a staircase function. The parameters of the radial-basis expansion are then updated using the standard procedure and thus stability is guaranteed.

The two significant advantages of modelling the nonlinearity in this way are that: i) there is no need to know nor estimate the functional form of the nonlinearity, and ii) adaptation can be applied to nonlinearities with highly complex features.

III. Dissipativity-Based Method

Feedback linearization algorithms are based on the idea of making the system linear and then design a linear controller to stabilize the linearized dynamics. However, the performances of this technique degrades quickly if the controller is unable to exactly cancel the nonlinearity, and even instability of the system can occur.

To overcome these difficulties and make the system more robust, a novel dissipativity based control method is investigated. This technique, as explained in [4], merges optimal nonlinearity approximation with dissipativity, to ensure that the closed loop dynamics remain stable even in presence of inexact nonlinarity cancellation. It is possible to rewrite Eq. (1) as

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u - \tilde{\mathbf{B}}\psi(z) \\ y = \mathbf{C}\mathbf{x} \\ z = \mathbf{D}\mathbf{x} \end{cases} \tag{11}$$

in which $z \in \mathbb{R}^{n_z}$ is the argument of the nonlinearity $\psi(\cdot)$.

An output feedback controller is designed in the form of

$$\begin{cases} \dot{\mathbf{v}} = \mathbf{R}\mathbf{v} + \mathbf{S}y \\ \omega = \mathbf{T}\mathbf{v} + \mathbf{U}y \longmapsto \begin{cases} \omega_1 = \mathbf{T_1}\mathbf{v} + U_1 y \\ \omega_2 = \mathbf{T_2}\mathbf{v} + U_2 y \end{cases} \\ u = -\omega_1 + \phi(\omega_2) \end{cases} \tag{12}$$

which is a generalization of the standard observer-based controller with a term $\phi(\omega_2)$ that cancels the nonlinearity. This yields the closed loop dynamics to be stable. The function $\phi(\cdot)$ represents the optimal approximation of the nonlinearity $\psi(\cdot)$ and it is obtained via a numerical minimization problem. Moreover, the controller (R, S, T, U) is defined by solving a linear matrix inequality, as also explained in [5].

IV. Aeroservoelastic Model

Here, the mathematical model is developed to simulate the control methods. Subsequently, the designed experimental rig is presented.

A. Numerical Model

The model consists of a rigid airfoil constrained to two degrees-of-freedom: a vertical translation h and a rotation around the elastic axis α, as shown in Fig. 1. By using Euler-Lagrange method it is possible to obtain the structural equation of motion

$$\begin{bmatrix} m_T & S_\alpha \\ S_\alpha & I_\alpha \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix} \begin{Bmatrix} \dot{h} \\ \dot{\alpha} \end{Bmatrix} + \begin{bmatrix} k_h(h) & 0 \\ 0 & k_\alpha \end{bmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} -L \\ M \end{Bmatrix} \tag{13}$$

where the structural fifth order polynomial nonlinearity is given by

$$k_h(h) = k_{h_0} + k_{h_2} h^2 + k_{h_4} h^4 \tag{14}$$

in the case of a geometric nonlinearity.

To describe the effect of the airflow on the airfoil we use Theodorsen's unsteady approximation for the aerodynamics loads and a quasi-steady approximation for the control. Therefore, lift L and moment M are given by

$$L = 2\pi\rho s_p U b C(k) \left(\dot{h} + U\alpha + b\left(\frac{1}{2} - a\right)\dot{\alpha}\right) + \\ + \pi\rho s_p b^2 \left(\ddot{h} + U\dot{\alpha} - ba\ddot{\alpha}\right) + \rho U^2 b \frac{s_p}{4}\left(C_{l_\beta}\beta + C_{l_\gamma}\gamma\right)$$

$$M = 2\pi\rho s_p U b^2 C(k)\left(\frac{1}{2} + a\right)\left(\dot{h} + U\alpha + b\left(\frac{1}{2} - a\right)\dot{\alpha}\right) + \\ + \pi\rho s_p b^3 \left(a\ddot{h} - \left(\frac{1}{2} - a\right)U\dot{\alpha} - b\left(\frac{1}{8} + a^2\right)\ddot{\alpha}\right) + \rho U^2 b^2 \frac{s_p}{4}\left(C_{m_\beta}\beta + C_{m_\gamma}\gamma\right) \tag{15}$$

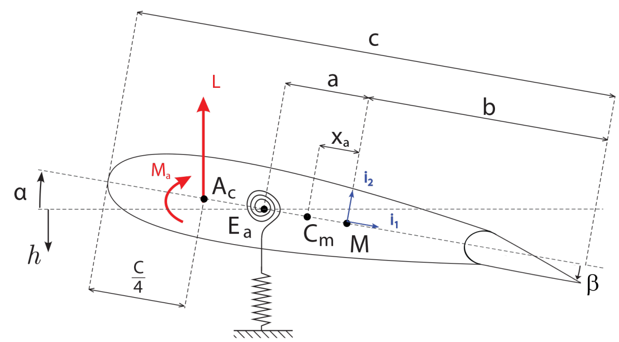


Fig. 1 Lateral view of the airfoil.

B. Experimental Model

To validate the 2 DOF mathematical model described in *Section II-A* and to check the effectiveness of the control technique developed, a physical setup of the system has been developed. All the experimental tests are performed in the wind tunnel of the University of Liverpool, which is able to generate a maximum wind speed of approximately 20 m/s.

The aerodynamic profile is a NACA 0018 and a trailing-edge control surface is mounted to the central sector, which covers 25% of the total span of 1.2 m. The flap can rotate up to ±10 degrees thanks to a 60W Maxon brush-less motors. The plunge hardening nonlinearity is physically reproduced using a clamped-clamped mild steel tensioned wire attached to the plunge DOF.

Both the external sector and the control surface have been realized by 3D printing in ABS. The support structure is in aluminum, the plunge spring is made of two parallel spring steel plates and the pitch spring is designed using a spring steel, as in Fig. 2.

In order to have a correct representation of the experimental system, identification of the model parameters is needed. Exciting the system with a known force F applied on the plunge DOF using a shaker and reading the response given by two lasers, it is possible to obtain the Frequency Response Functions (FRFs) of the system.

Then, the identification of the parameters is made fitting the analytical transfer function to the experimental one. The initial stiffness k_h and k_α are directly measured by experimental tests and the other parameters are initially set to zero. This is possible because least squares minimization is a global optimization and there is no possibility to stop in a local minimum. Fig. 3 shows some preliminary results of the comparison between the experimental FRF and the optimized analytical FRF.

V. Preliminary Results

A. Adaptive Feedback Linearization using Radial-Basis Functions

The method has been tested on the numerical aeroservoelastic system with a freeplay nonlinearity in the plunge degree-of-freedom of the form

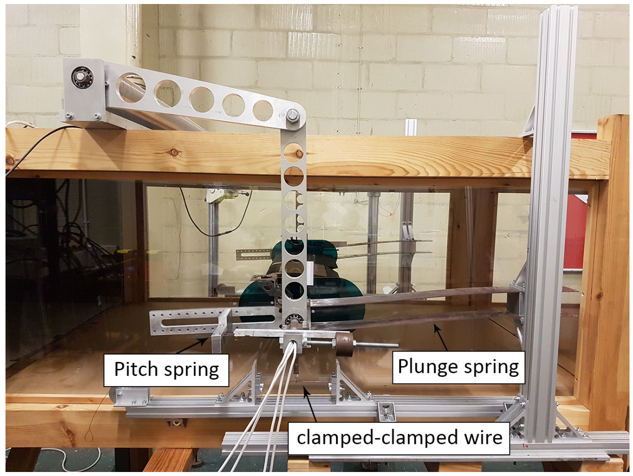
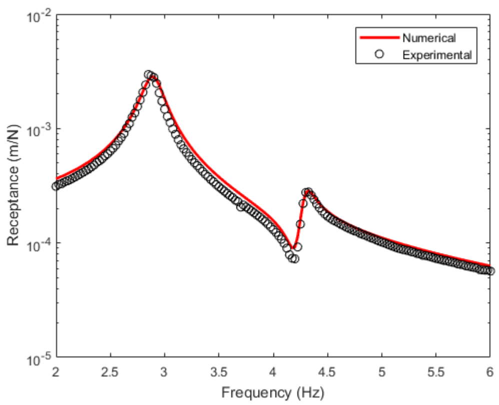
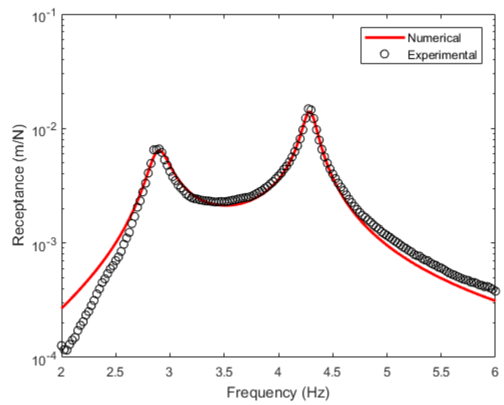


Fig. 2 Experimental setup.



(a) *Plunge FRFs*



(b) *Pitch FRFs*

Fig. 3 The red curve is the analytical approximation with the optimized parameters, the circles represent the experimental FRF.

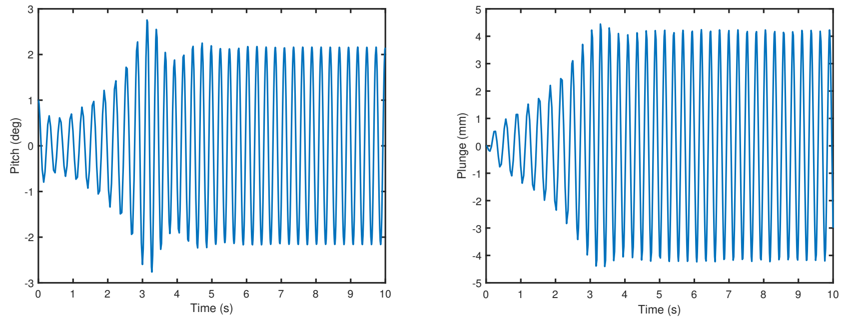
$$f_h(h) = f_{\mathrm{LN}}(h) + f_{\mathrm{NL}}(h) = (-H(-h - 0.03) + H(h - 0.03)) \times 2.4 \times 10^3 h + k_h h \tag{16}$$

where $H(.)$ denotes the Heaviside function. The system exhibits a limit cycle at v = 13 m/s with an amplitude of 2.2 degrees and 4.2 mm in the pitch and plunge degrees-of-freedom, respectively, as illustrated in Fig. 4 (a) and (b).

The controller is designed using 22 radial basis functions split evenly across the range $x = \pm 5$ mm. Initially, all of the weighting constants of the nonlinearity are set to zero and thus there is no cancellation of the nonlinearity nor the linear part of the plunge stiffness. After 5 seconds, the controller and parameter updating is switched on. Figure 5 (a) and (b) shows the response of the pitch and plunge degrees-of-freedom and Fig. 6 shows the time history of the parameters. After approximately 1.5 seconds, the parameters are shown to converge and seemingly approximate the nonlinearity well.

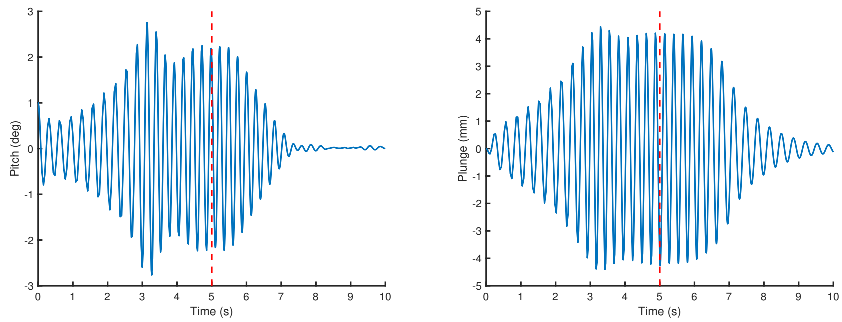
B. Dissipativity-Based Method

In Fig. 7-8 is possible to see the preliminary results of the comparison between feedback linearization and dissipativity. We can see that both techniques successfully stabilize the nonlinear behaviour and suppress the LCO.



(a) *Pitch displacement.* (b) *Plunge displacement.*

Fig. 4 Open-loop time series for the non-smooth nonlinearity



(a) *Pitch displacement.* (b) *Plunge displacement.*

Fig. 5 Control-loop time series for the non-smooth nonlinearity

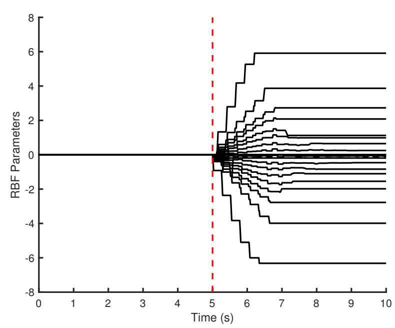


Fig. 6 Parameter update and estimation

VI. Conclusions

Preliminary results show that both techniques are able to suppress LCOs when either the parameters or the functional-form of the nonlinearty is unknown. The radial-basis adaptive method is able to entirely suppress the LCO, even when the functional form of the nonlinearity is completely unknown or discontinuous. The dissipative method is able to stabilize the system in less time than conventional feedback linearization, but at the expense of a higher control action on the control surface. In the final version of this paper, both methods will be implemented on the physical system in order to validate the predicted behavior and performance, and to confirm their applicability to real aeroelastic systems.

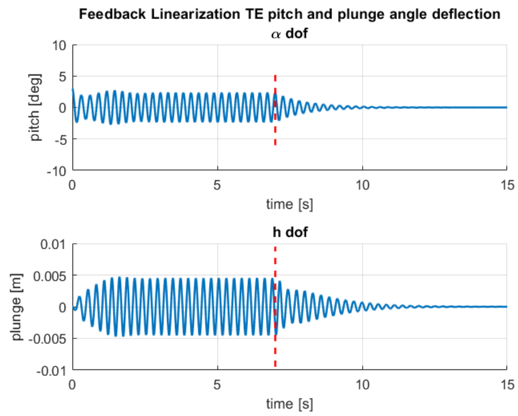


Fig. 7 Closed loop response using feedback linearization control with pole placement.

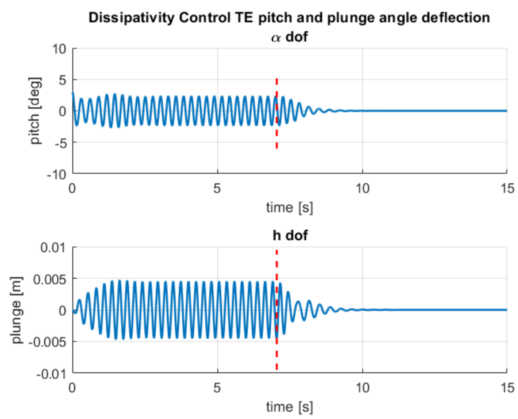


Fig. 8 Closed loop response using dissipativity based control.

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